arXiv:nucl-th/0111077 v1 28 Nov 2001

Relativistic calculation of nuclear transparency in $(e, e'p)$ reactions

Andrea Meucci
Dipartimento di Fisica Nucleare e Teorica, Università di Pavia and
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, Italy
(Dated: January 30, 2003)

Nuclear transparency in $(e, e'p)$ reactions is evaluated in a fully relativistic distorted wave impulse approximation model. The relativistic mean field theory is used for the bound state and the Pauli reduction for the scattering state, which is calculated from a relativistic optical potential. Results for selected nuclei are displayed in a Q^2 range between 0.3 and 1.8 $(\mathrm{GeV}/c)^2$ and compared with recent electron scattering data. For $Q^2 = 0.3$ $(\mathrm{GeV}/c)^2$ the results are lower than data; for higher Q^2 they are in reasonable agreement with data. The sensitivity of the model to different prescriptions for the one-body current operator is investigated. The off-shell ambiguities are rather large for the distorted cross sections and small for the plane wave cross sections.

PACS numbers: 25.30.Fj, 24.10.Jv

I. INTRODUCTION

Exclusive $(e, e'p)$ knockout reactions have been used since a long time to study the single particle properties of nuclear structure. The analysis of the experimental cross sections were successfully carried out in the theoretical framework of the nonrelativistic distorted wave impulse approximation (DWIA) for Q^2 less than 0.4 GeV/c^2 [1, 2]. In recent years, owing to the new data from TJNAF [3, 4], similar models based on a fully relativistic DWIA (RDWIA) framework were developed. In this approach the wave functions of the initial and final nucleons are solutions of a Dirac equations containing scalar and vector potentials fitted to the ground state properties of the nucleus and to proton elastic scattering data [5].

In the nucleus, final state interaction with the nuclear medium can absorb the struck proton, thus reducing the experimental cross section. This reduction is related to the nuclear transparency, which can be intuitively defined as the ratio of the measured to the plane wave cross section. The transparency can be used to refine our knowledge of nuclear medium effects and to look for deviation from conventional predictions of nuclear physics, such as the Color Transparency (CT) effect. The CT was introduced basing on perturbative QCD arguments [6]. The name is related to the disappearance of the color forces at high Q^2: three quarks should form an object that passes through the nuclear medium without undergoing interactions. If the CT effect switches on as Q^2 increases, then the nuclear transparency should be enhanced towards unity. Several measurements of the nuclear transparency in $(p, 2p)$ and $(e, e'p)$ knockout have been carried out in the past. The first experiment looking for CT effect was performed at Brookhaven [7] measuring transparency in $(p, 2p)$ reaction. An increase of transparency for $3 \le Q^2 \le 8$ $(\mathrm{GeV}/c)^2$, followed by a decrease for $8 \le Q^2 \le 11$ $(\mathrm{GeV}/c)^2$ was observed. New data confirm this energy dependence of transparency [8]. The first measurements of nuclear transparency in $(e, e'p)$ reaction were carried out at Bates with $Q^2 = 0.3$ $(\mathrm{GeV}/c)^2$ [9]. In recent years, higher energy data of transparency in $(e, e'p)$ were produced at SLAC [10] and TJNAF [11, 12]. In contrast with $(p, 2p)$ data, the NE-18 experiment at SLAC did not see any CT effect up to $Q^2 = 6.8$ $(\mathrm{GeV}/c)^2$, but could not exclude a slow onset of CT. The E91-013 experiment at TJNAF studied the nuclear transparency in a Q^2 range up to 8.1 $(\mathrm{GeV}/c)^2$ with greatly improved statistics and did not found evidence for the onset of CT.

The distorted wave approach was first applied to evaluate transparency in $(e, e'p)$ knockout in Ref. [13], where it was shown that measurements of the normal transverse structure function in $^{208}\mathrm{Pb}$ could afford to see CT effect, and in Ref. [14], where the nuclear part of the transition amplitude was written in terms of Schrödinger-like wave functions for bound and scattering states and of an effective current operator containing the Dirac potentials. Alternatively, the nuclear transparency results were analyzed in terms of a Glauber model [15, 16, 17], which assumes classical attenuation of protons in the nuclear medium.

In this paper we present RDWIA calculations of nuclear transparency in $(e, e'p)$ reaction. The RDWIA treatment is the same as in Refs. [18, 19]. The relativistic bound state wave functions have been generated as solutions of a Dirac equation containing scalar and vector potentials obtained in the framework of the relativistic mean field theory. The effective Pauli reduction has been adopted for the outgoing nucleon wave function. The resulting Schrödinger-like equation is solved for each partial wave starting from relativistic optical potentials. The relativistic current is written following the most commonly used current conserving (cc) prescriptions for the $(e,e'p)$ reaction introduced in Ref. [20]. The ambiguities connected with different choices of the electromagnetic current cannot generally be dismissed. In the $(e,e'p)$ reaction the predictions of different prescriptions are generally in close agreement [21]. Large differences can however be found at high missing momenta [22, 23].

The formalism is outlined in Sec. II. Relativistic calculations of nuclear transparency are presented in Sec. III, where current ambiguities are also investigated. Some conclusions are drawn in Sec. IV.

II. FORMALISM

The nuclear transparency can be experimentally defined as the ratio of the measured cross section to the cross section in plane wave approximation, which is usually evaluated by means of a Monte Carlo simulation to take in account the kinematics of the experiment. Hence, we define nuclear transparency as

$$T = \frac{\int_V \mathrm{d}E_\mathrm{m}\,\mathrm{d}\boldsymbol{p_\mathrm{m}}\;\sigma_{DW}\left(E_\mathrm{m},\boldsymbol{p_\mathrm{m}},\boldsymbol{p'}\right)}{\int_V \mathrm{d}E_\mathrm{m}\,\mathrm{d}\boldsymbol{p_\mathrm{m}}\;\sigma_{PW}\left(E_\mathrm{m},\boldsymbol{p_\mathrm{m}}\right)}\ , \tag{1}$$

where σ_{DW} is the distrorted wave cross section and σ_{PW} is the plane wave one. Since the measured transparency depends upon the kinematics conditions and the spectrometer acceptance, we have to specify the space phase volume, V, and use it for both the numerator and the denominator [24]. Because of final state interaction, the distorted cross section depends upon the momentum of the emitted nucleon $\boldsymbol{p'}$, whereas the undistorted cross section only depends upon the missing energy E_m and the missing momentum $\boldsymbol{p_\mathrm{m}}$.

In the one-photon exchange approximation the $(e,e'p)$ cross section is given by the contraction between the lepton tensor and the hadron tensor. In the case of an unpolarized reaction it can be written as

$$\sigma = \sigma_\mathrm{M}\ f_\mathrm{rec}\ E'|\boldsymbol{p'}|\ \left[\rho_{00}f_{00} + \rho_{11}f_{11} + \rho_{01}f_{01}\cos\left(\alpha\right) + \rho_{1-1}f_{1-1}\cos\left(2\alpha\right)\right]\ , \tag{2}$$

where σ_M is the Mott cross section, f_rec is the recoil factor [1, 2], E' and $\boldsymbol{p'}$ are the energy and momentum of the emitted nucleon, and α is the out of plane angle between the electron scattering plane and the $(\boldsymbol{q},\boldsymbol{p'})$ plane. The coefficients $\rho_{\lambda\lambda'}$ are obtained from the lepton tensor components and depend only upon the electron kinematics [1, 2]. The structure functions $f_{\lambda\lambda'}$ are given by bilinear combinations of the components of the nuclear current as

$$\begin{aligned}
f_{00} &= \langle J^0\left(J^0\right)^\dagger\rangle\ ,\\
f_{11} &= \langle J^x\left(J^x\right)^\dagger\rangle + \langle J^y\left(J^y\right)^\dagger\rangle\ ,\\
f_{01} &= -2\sqrt{2}\,\mathfrak{Re}\left[\langle J^x\left(J^0\right)^\dagger\rangle\right]\ ,\\
f_{1-1} &= \langle J^y\left(J^y\right)^\dagger\rangle - \langle J^x\left(J^x\right)^\dagger\rangle\ ,
\end{aligned} \tag{3}$$

where $\langle\cdots\rangle$ means that average over the initial and sum over the final states is performed fulfilling energy conservation. In our frame of reference the z axis is along $\boldsymbol{q}$, and the y axis is parallel to $\boldsymbol{q}\times\boldsymbol{p'}$.

In RDWIA the matrix elements of the nuclear current operator, i.e.,

$$J^\mu = \int \mathrm{d}\boldsymbol{r}\overline{\Psi}_f(\boldsymbol{r})\hat{j}^\mu\exp\{i\boldsymbol{q}\cdot\boldsymbol{r}\}\Psi_i(\boldsymbol{r})\ , \tag{4}$$

are calculated using relativistic wave functions for initial and final states.

The choice of the electromagnetic operator is a longstanding problem. Here we discuss the three current conserving expressions [20, 25, 26]

$$
\begin{aligned}
\hat{j}^{\mu}_{cc1} &= G_M(Q^2)\gamma^{\mu} - \frac{\kappa}{2M} F_2(Q^2)\overline{P}^{\mu} \;, \\
\hat{j}^{\mu}_{cc2} &= F_1(Q^2)\gamma^{\mu} + i\frac{\kappa}{2M} F_2(Q^2)\sigma^{\mu\nu}q_{\nu} \;, \\
\hat{j}^{\mu}_{cc3} &= F_1(Q^2)\frac{\overline{P}^{\mu}}{2M} + \frac{i}{2M} G_M(Q^2)\sigma^{\mu\nu}q_{\nu} \;,
\end{aligned}
\tag{5}
$$

where $q^{\mu} = (\omega, \boldsymbol{q})$ is the four-momentum transfer, $Q^2 = \mid \boldsymbol{q} \mid^2 - \omega^2$, $\overline{P}^{\mu} = (E + E', \boldsymbol{p}_{\mathbf{m}} + \boldsymbol{p}')$, κ is the anomalous part of the magnetic moment, F_1 and F_2 are the Dirac and Pauli nucleon form factors, $G_M = F_1 + \kappa F_2$ is the Sachs nucleon magnetic form factor, and $\sigma^{\mu\nu} = (i/2)\,[\gamma^{\mu}, \gamma^{\nu}]$. These expressions are equivalent for on-shell particles thanks to Gordon identity. However, since nucleons in the nucleus are off-shell we expect that these formulas should give different results. Current conservation is restored by replacing the longitudinal current and the bound nucleon energy by [20]

$$
J^L = J^z = \frac{\omega}{\mid \boldsymbol{q} \mid} J^0 \;, \tag{6}
$$

$$
E = \sqrt{\mid \boldsymbol{p}_{\mathbf{m}} \mid^2 + M^2} = \sqrt{\mid \boldsymbol{p}' - \boldsymbol{q} \mid^2 + M^2} \;. \tag{7}
$$

The bound state wave function

$$
\Psi_i = \begin{pmatrix} u_i \\ v_i \end{pmatrix} \;, \tag{8}
$$

is given by the Dirac-Hartree solution of a relativistic Lagrangian containing scalar and vector potentials.

The ejectile wave function Ψ_f is written in terms of its positive energy component Ψ_{f+} following the direct Pauli reduction method [27]

$$
\Psi_f = \begin{pmatrix} \Psi_{f+} \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}'}{M+E'+S-V}\Psi_{f+} \end{pmatrix} \;, \tag{9}
$$

where $S = S(r)$ and $V = V(r)$ are the scalar and vector potentials for the nucleon with energy E'. The upper component Ψ_{f+} is related to a Schrödinger equivalent wave function Φ_f by the Darwin factor $D(r)$, i.e.,

$$
\Psi_{f+} = \sqrt{D(r)}\Phi_f \;, \tag{10}
$$

$$
D(r) = \frac{M + E' + S - V}{M + E'} \;. \tag{11}
$$

Φ_f is a two-component wave function which is solution of a Schrödinger equation containing equivalent central and spin-orbit potentials obtained from the scalar and vector potentials. Hence, using the relativistic normalization, the emitted nucleon wave function is written as

$$
\begin{aligned}
\overline{\Psi}_f = \Psi_f^{\dagger}\gamma^0 &= \sqrt{\frac{M + E'}{2E'}} \left[\begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\boldsymbol{p}'}{C} \end{pmatrix} \sqrt{D}\;\Phi_f \right]^{\dagger} \gamma^0 \\
&= \sqrt{\frac{M + E'}{2E'}}\; \Phi_f^{\dagger} \left(\sqrt{D}\right)^{\dagger} \left(1 \;;\; \boldsymbol{\sigma}\cdot\boldsymbol{p}'\frac{1}{C^{\dagger}}\right)\gamma^0 \;,
\end{aligned}
\tag{12}
$$

where

$$
C = C(r) = M + E' + S(r) - V(r) \;. \tag{13}
$$

III. TRANSPARENCY AND THE $(e, e'p)$ REACTION

The $(e, e'p)$ reaction is a well-suited process to search for CT effects. The e-p cross section is accurately known from QED and the energy resolution guarantees the exclusivity of the reaction.

Several measurements of nuclear transparency to protons in quasifree $(e, e'p)$ knockout have been carried out on several target nuclei and over a wide range of energies to look for CT onset.

Here, we calculated nuclear transparency for closed shell or subshell nuclei at kinematics conditions compatible with the experimental setups for which the measurements of nuclear transparency have been performed, and for which the RDWIA predictions are known to provide a good agreement with cross section data. The bound state wave functions and optical potentials are the same as in Refs. [18, 19], where the RDWIA results are in satisfactory agreement with $(e, e'p)$ and (γ, p) data.

The relativistic bound-state wave functions have been obtained from the program ADFX of Ref. [28], where relativistic Hartree-Bogoliubov equations are solved in the mean field approximation to the description of ground state properties of several spherical nuclei. The model starts from a Lagrangian density containing sigma-meson, omega-meson, rho-meson and photon field, whose potentials are obtained by solving self-consistently Klein-Gordon equations. Moreover, finite range interactions are included to describe pairing correlations and the coupling to particle continuum states.

The outgoing nucleon wave function is calculated by means of the complex phenomenological optical potential EDAD1 of Ref. [29], which is obtained from fits to proton elastic scattering data on several nuclei in an energy range up to 1040 MeV.

Since no rigorous prescription exists for handling off-shell nucleons, we have studied the sensitivity to different cc choices of the nuclear current. The Dirac and Pauli form factors F_1 and F_2 are taken from Ref. [30].

In Fig. 1 our RDWIA results for nuclear transparency, calculated with the $cc2$ prescription for the nuclear current are shown. The Q^2 of the exchanged photon is taken between 0.3 (GeV$/c)^2$ and 1.8 (GeV$/c)^2$ in constant $(\boldsymbol{q}, \omega)$ kinematics. Calculations have been performed for selected closed shell or subshell nuclei (^{12}C, ^{16}O, ^{28}Si, ^{40}Ca, ^{90}Zr, and ^{208}Pb) for which the relativistic mean field code easily converges. The agreement with the data is rather satisfactory. At $Q^2 = 0.3$ (GeV$/c)^2$ our results lie below the data and are comparable with those presented in Ref. [14], where it was shown that the EDAD1 optical potential led to a smaller transparency, while better agreement was found using an empirical effective interaction which fits both proton elastic and inelastic scattering data. However, we have to note that the DWIA model of Ref. [14] uses a different approach to obtain single particle bound state wave functions. The calculations at $Q^2 = 0.6, 1.3,$ and 1.8 (GeV$/c)^2$ are closer to the data and fall down only for higher mass numbers.

In Fig. 2 the energy dependence of nuclear transparency is shown. The calculations have been performed for the same nuclei and at the same kinematics as in Fig. 1. The calculated transparency is approximately constant for each nucleus and decreases for increasing mass number.

In Refs. [10, 12] it is reported that the transparency data can be fitted with an exponential law of the form $T = A^{-\alpha}$, with $\alpha \simeq 0.24$. Since our model is based on a single particle picture of nuclear structure, we expect our results to be sensible to the discontinuities of the shell structure. These clearly appear in the changes in shape of the A-dependent curves.

In Fig. 3 the sensitivity of transparency calculations for ^{12}C and ^{40}Ca to different choices for the electromagnetic current is shown. The results with the $cc1$ current are larger than those obtained with the $cc2$ current, whereas $cc3$ results are smaller than the $cc2$ ones. A similar behavior was already found out in Ref. [19] for (γ, N) differential cross section. Here it is mainly due to the fact that, when using the $cc1$ current, the distorted cross section, σ_{DW} in Eq. 1, is enhanced with respect to the calculations with the $cc2$ or the $cc3$ current, whereas the plane wave cross sections, σ_{PW}, are almost independent of the operator form.

IV. SUMMARY AND CONCLUSIONS

In this paper we have presented relativistic DWIA calculations for nuclear transparency of $(e, e'p)$ reaction in a momentum transfer range between 0.3 and 1.8 (GeV$/c)^2$.

The transition matrix element of the nuclear current operator in RDWIA is calculated using the bound state wave functions obtained in the framework of the relativistic mean field theory, and the direct Pauli reduction method with scalar and vector potentials for the scattering state. In order to analyze the ambiguities in the choice of the electromagnetic vertex due to the off-shell character of the initial nucleon, we have used three current conserving expressions in our calculations.

We have performed calculations for selected closed shell or subshell nuclei. The dependence of nuclear transparency upon the mass number and the energy has been discussed. Low Q^2 results underestimates the data, thus indicating the presence of too strong an absorptive term in the optical potential. In contrast, results at higher Q^2 are closer to the data. We find little evidence of energy dependence or momentum transfer of the transparency for each nucleus.

The sensitivity to different choices of the nuclear current has been investigated for ^{12}C and ^{40}Ca. The results with the $cc1$ current are larger than the $cc2$ results, whereas those obtained with the $cc3$ current are more similar to the $cc2$ ones. This effect is due to the enhancement of the $cc1$ distorted cross section with respect to the $cc2$ and $cc3$ cross sections.

Acknowledgments

I would like to thank Professor C. Giusti and Professor F. D. Pacati for useful discussions and comments on the manuscript.

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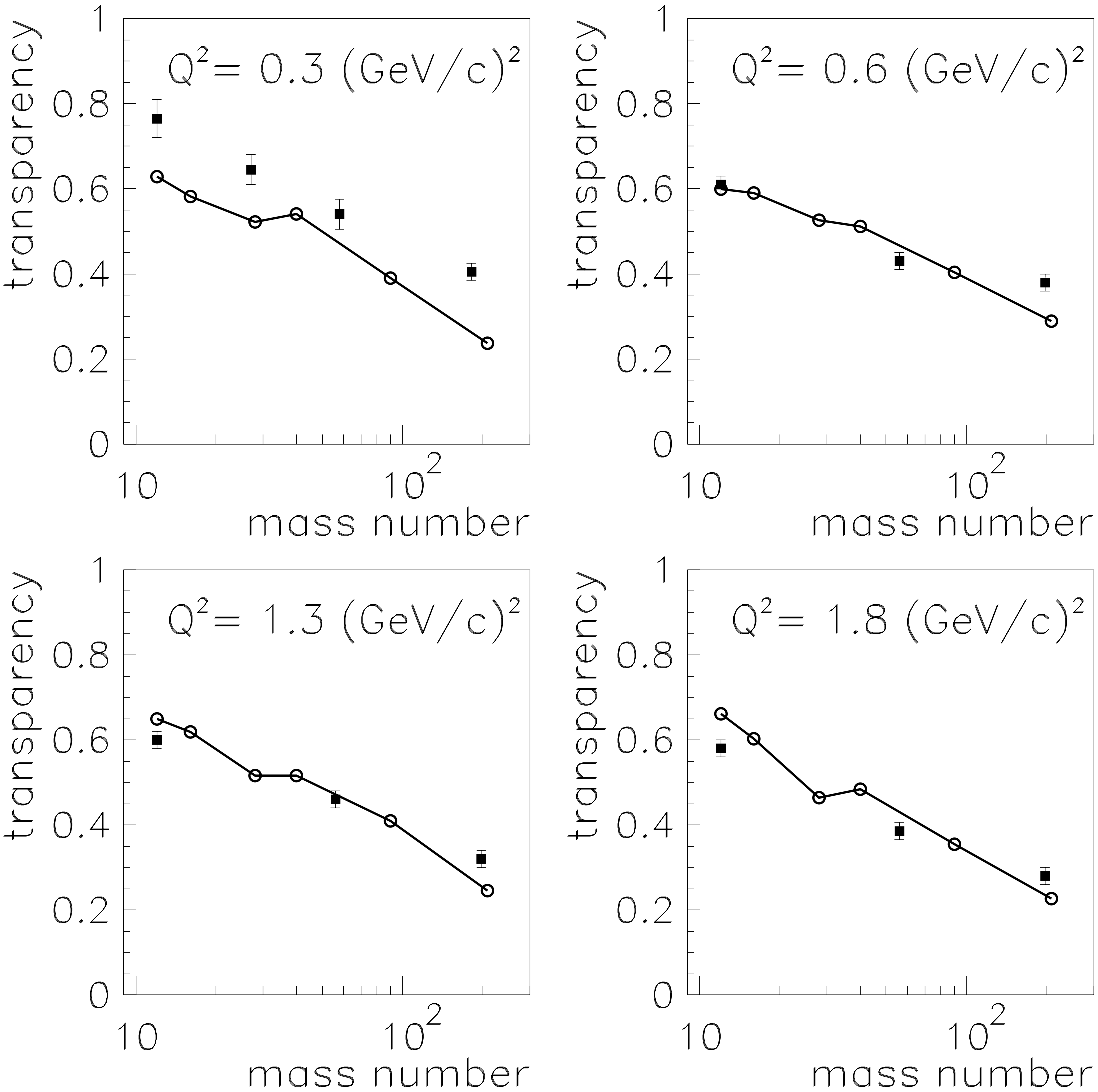

FIG. 1: The nuclear transparency for the quasifree A$(e,e'p)$ reaction as a function of the mass number for Q^2 ranging from 0.3 to 1.8 (GeV/c)^2. Calculations have been performed for selected closed shell or subshell nuclei with mass numbers indicated by open circles. The data at $Q^2 = 0.3$ (GeV/c)^2 are from Ref. [9]. The data at $Q^2 = 0.6, 1.3,$ and 1.8 (GeV/c)^2 are from Ref. [11].

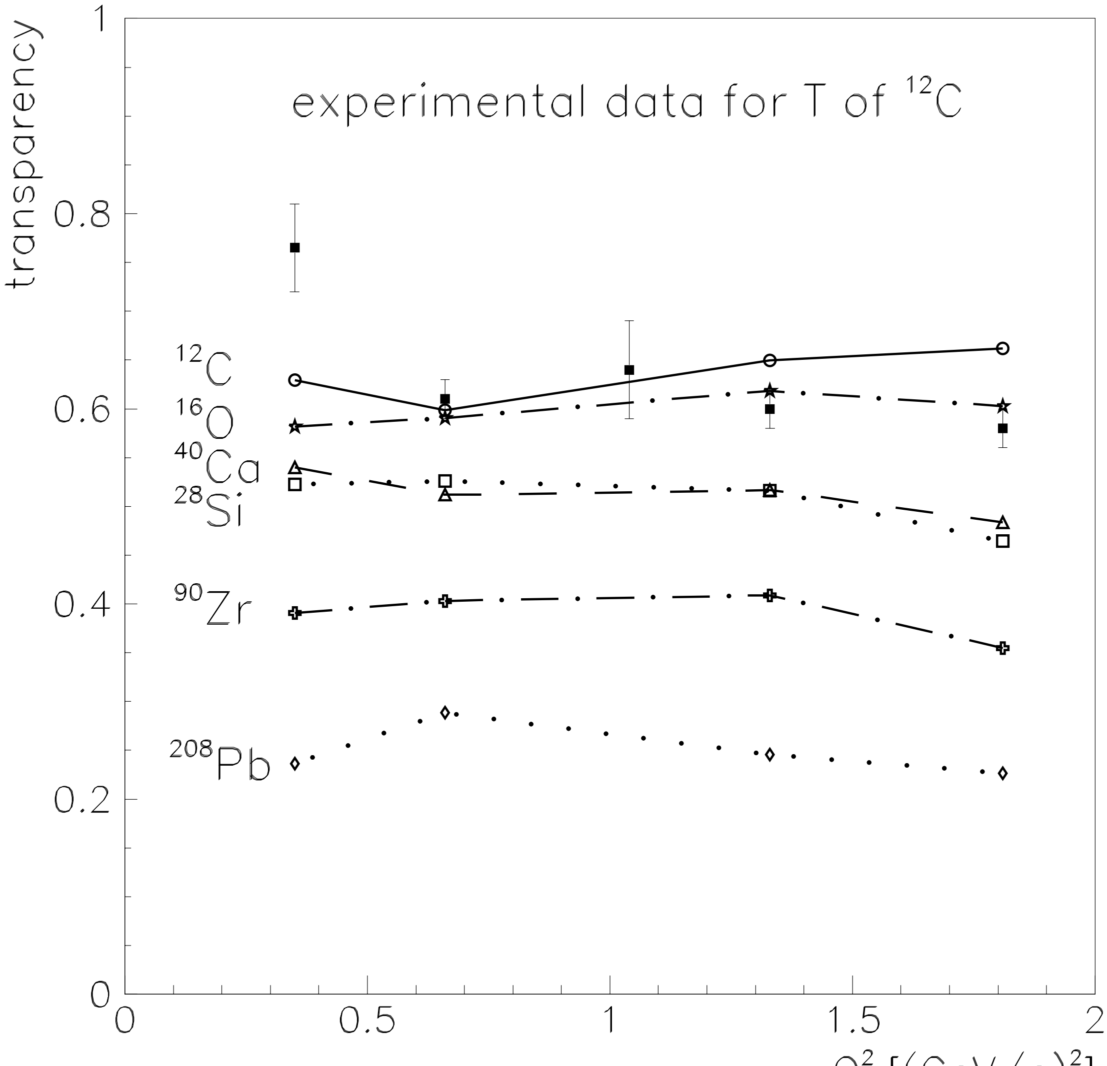

FIG. 2: The energy dependence of nuclear transparency for ^{12}C (open circles), ^{16}O (open stars), ^{28}Si (open squares), ^{40}Ca (open triangles), ^{90}Zr (open crosses), and ^{208}Pb(open diamonds), at the same kinematics as in Fig. 1. Calculations were performed for Q^2 values marked by symbols. The ^{12}C data are from Refs. [9, 10, 11].

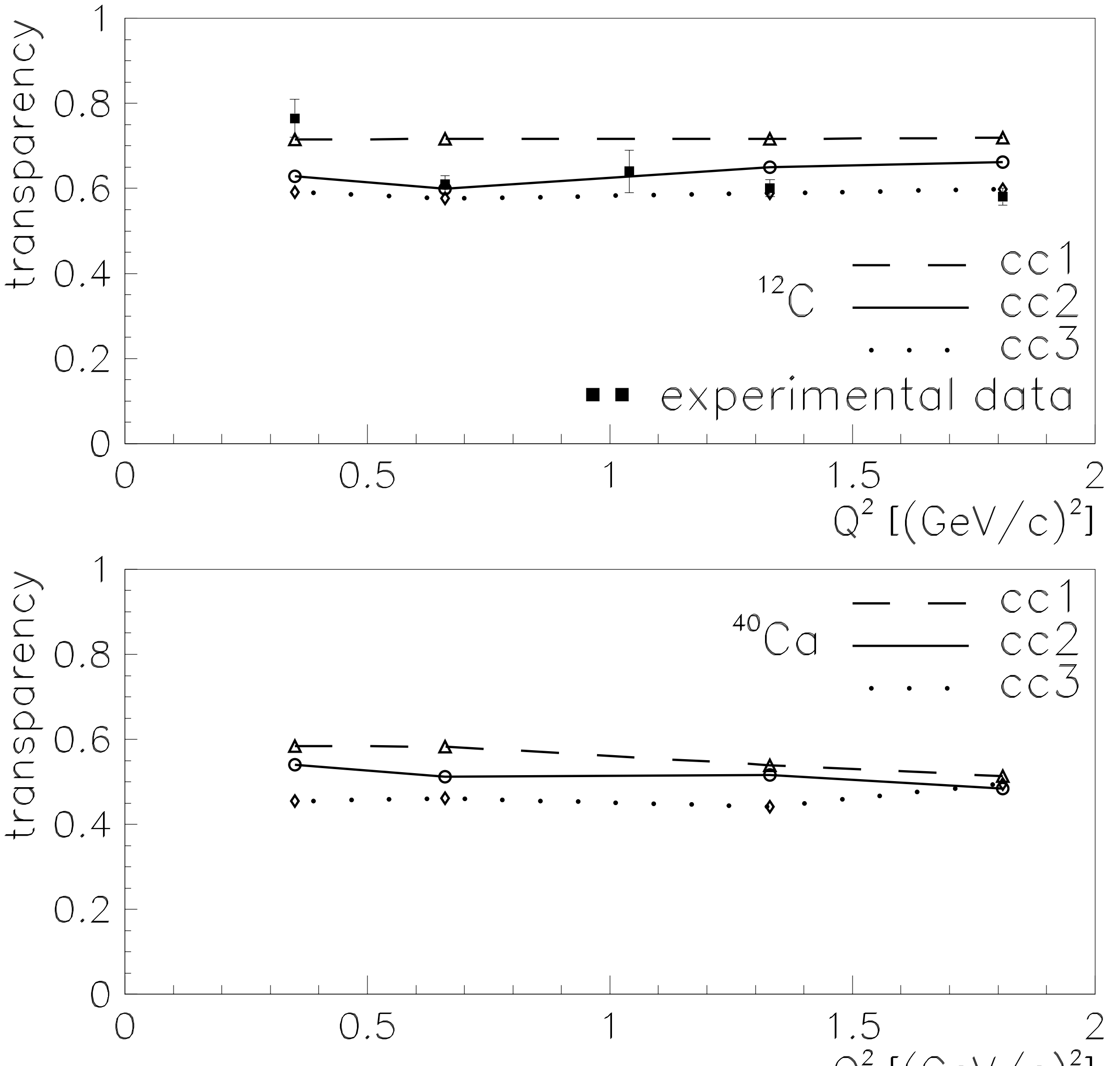

FIG. 3: The electromagnetic current dependence of nuclear transparency for ^{12}C and ^{40}Ca, at the same kinematics as in Fig. 1. Calculations were performed for Q^2 values marked by symbols.